Delayed scattering of solitary waves from interfaces in a granular container

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In granular media, the characterization of the behavior of solitary waves around interfaces is of importance in order to look for more applications of these systems. We study the behavior of solitary waves at both interfaces of a symmetric granular container, a class of systems that has received recent attention because it possesses the feature of energy trapping. Hertzian contact is assumed. We have found that the scattering process is elastic at one interface, while at the other interface it is observed that the transmitted solitary wave has stopped its movement during a time that gets longer when the ratio between masses at the interfaces increases. The origin of this effect can be traced back to the phenomenon of gaps opening, recently observed experimentally.

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The propagation of a perturbation in a chain of beads in Hertzian contact possesses soliton-like features, as first observed by Nesterenko [1]. Several studies, experimental [2,3] as well theoretical [4,5] have confirmed the existence of such soliton-like pulses. Despite the large amount of recent work on the subject [3,5–16], the physics of granular media remains a challenge and new effects are there to be discovered and studied. Enlarging the number of (engineering) applications of such new effects needs a complete understanding of the dynamics of such granular media.

The simplest granular systems are one-dimensional chains of elastic spheres. If they are in Hertzian contact, the spheres may be considered as point masses interacting through massless nonlinear springs with elastic force $F=k\delta^{3/2}$, where δ is the overlap of contacts and k is the spring constant (a function of the material properties) [1]. Let $x_i(t)$ represent the displacement of the center of the ith sphere from its initial equilibrium position, and assume that the ith sphere, of mass m_i, has neighbors of different radii (and/or mechanical properties). Then, in absence of load and in a frictionless medium, the equation of motion for the ith sphere reads

$$m_i \frac{d^2 x_i}{dt^2} = k_1 (x_{i-1} - x_i)^{3/2} - k_2 (x_i - x_{i+1})^{3/2}, \tag{1}$$

where it is understood that the brackets take the argument value if they are positive and zero otherwise, ensuring that the spheres interact only when in contact.

The interaction of a solitary wave with the boundary of two "sonic vacua" (meaning that the system does not support linear sound waves if not precompressed) was studied for the first time experimentally as well numerically in [5] (see also [17–20] for a recent study).

In this work we make a detailed numerical study of the propagation of solitary waves in a linear chain of beads composed of three sonic vacua, as shown in Fig. 1, that is, a granular container [21,22]. These kinds of systems are of interest because in them one can find the phenomenon of energy trapping. It will be assumed that all spheres have the same mechanical properties and that both ends of the chain are free to move. We have found that the scattering process is elastic at one interface, while at the other interface it is observed that the transmitted solitary waves take a long time to be released from one of the interfaces; this time gets longer when the ratio between masses at the interfaces increases. The origin of this effect can be traced back to the phenomenon of gaps opening, recently observed experimentally [17,18]. As far as we know, the effect found here has not yet been observed in experiments.

Consider a set of spheres with two different radii a and b. It is known that adjacent spheres of radii a and b will interact with a force $F=k_{ab}\delta^{3/2}$, where

$$k_{ab} = \frac{\sqrt{ab/(a+b)}}{2\theta}, \tag{2}$$

with

$$\theta = \frac{3(1-\nu^2)}{4E} \tag{3}$$

and E is the Young modulus and ν the Poisson ratio of the bead material.

We will consider the scattering of solitary waves in a system like the one of Fig. 1, consisting of a total of M beads. There are two sets of beads with N_1 beads located on the left hand side, N_2 on the right hand side; both sets have beads with radii a and masses m_1. Between them there are L beads with radii b $(a>b)$ and masses m_2. Bead displacements are governed by a set of equations of motion that can be readily obtained from the successive application of Eq. (1), having in mind that the equation of motion for the first (resp. the last) sphere only includes the second (resp. the first) term, in case when there is no wall (as we here assume). Spring constants are k_{bb} in the middle, k_{aa} at right and left hand sides and k_{ab} at the interfaces of the granular system.

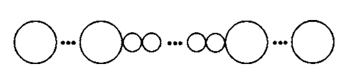

FIG. 1. Schematic granular container used in the calculations.

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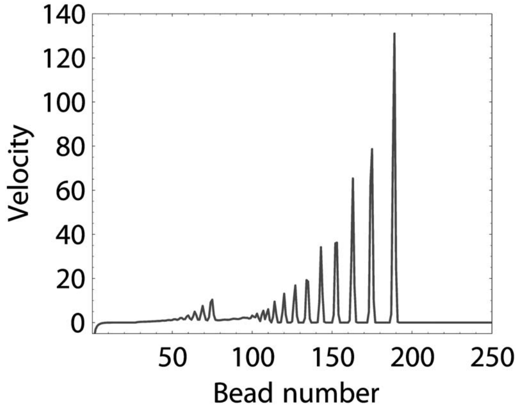

FIG. 2. Velocity of beads (in program units) as a function of bead number. Primary and secondary multipulse structures.

In order to have realistic results, we shall assume that the system consists of stainless-steel beads (see [3] for their properties), with radii $a=4$ mm and $b=2$ mm. The number of beads is $N_1=30$, $N_2=20$ and $L=200$. We also choose $\beta=10^{-5}$ m, 2.36×10^{-5} kg and $\alpha=1.0102\times10^{-3}$ s as units of distance, mass and time, respectively. Throughout the paper we assume that initially all beads are at rest, except for the first bead at the left side of the chain. This bead is supposed to have a nonzero value of velocity in order to generate the soliton-like perturbation in the chain. We shall choose the following initial conditions:

$$u_i(0)=0, \quad i=1,\ldots,M, \quad \dot{u}_1(0)=101.02\beta/\alpha,$$

$$\dot{u}_i(0)=0, \quad i=2,\ldots,M.$$

This initial impact velocity corresponds to 1 m/s and, therefore, it is in the regime where plastic deformation can be neglected. The system is studied numerically by using an explicit Runge-Kutta method of fifth order based on the Dormand-Prince coefficients, with local extrapolation. As step size controller we have used the proportional-integral step control, which gives a smooth step size sequence.

As the solitary wave gets the interface a multipulse structure is generated but no backscattered solitary wave is observed. This last phenomenon has been explained by Nesterenko *et al.* [17] as due to the opening of gaps in the vicinity of left interface. These effects originate from the discreteness of inertia and the nonlinearity of the interaction; they were first observed by Nesterenko and co-workers [5].

As the multipulse structure moves into the light system, there remains some energy behind the interface and after a while a second multipulse structure emerges, with similar characteristics to the first one but with less energy [20]. This is shown in Fig. 2. (It can be shown that the opening of gaps in the vicinity of the left interface is also responsible for the emergence of this structure.)

When the first multipulse structure interacts with the second interface it gets immediately scattered and both transmitted and reflected train of pulses appear in the right heavy system and in the light system, respectively. Part of this process is shown in Fig. 3.

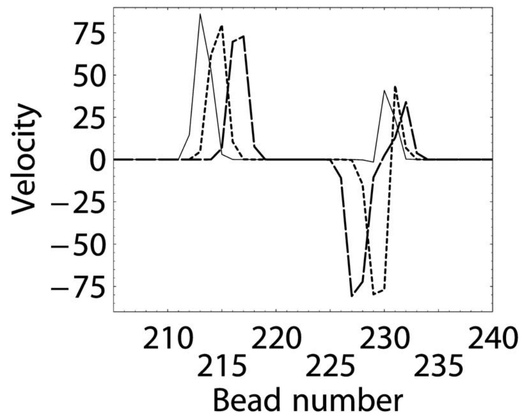

FIG. 3. Velocity of beads (in program units) as a function of bead number. Scattering of the leading pulse of the multipulse structure at the second interface, for different times: $t=1.303\ \alpha$ (full line), $t=1.31\ \alpha$ (dashed line) and $t=1.32\ \alpha$ (long-dashed line).

Now let us see what happens when the backscattered pulses (see Fig. 4) interact with the left boundary of the granular potential well. It is observed that at time $t=2.32\ \alpha$ the leading backscattered pulse arrives at the interface. Contrary to what happens at the right interface, the transmitted pulse moves slightly forward till time $t=2.343\ \alpha$ where it "freezes." Even more intriguing is the fact that only beyond time $t=2.462\ \alpha$ does the transmitted pulse starts to move into the heavy system. In our original units of time this means that the transmitted pulse has stopped its movement during 1.2×10^{-4} s approximately. This coincides with the fact that the second pulse of the multipulse structure approaches close to the interface. It is interesting to notice also that the backscattered pulses scatter without delay at this interface.

If we allow for the first bead to have more energy at $t=0$ s, it is observed that the characteristics of the scattering process around the second interface are similar to the one observed before: the scattering is elastic. At the left interface the situation has not essentially changed; there still is a delayed transmitted pulse.

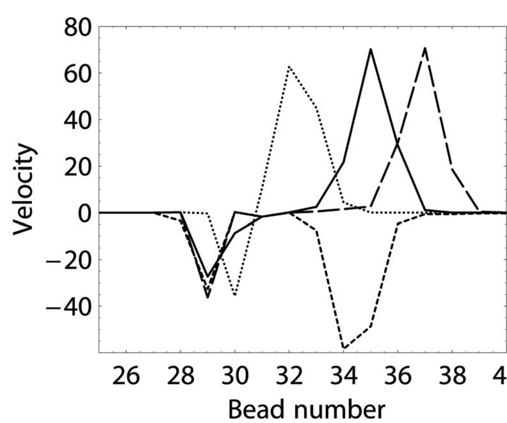

FIG. 4. Velocity of beads (in program units) as a function of bead number. Scattering of the first pulse of the backscattered multipulse structure at the first interface, for different times: $t=2.335\ \alpha$ (dotted line), $t=2.35\ \alpha$ (full line), $t=2.36\ \alpha$ (long-dashed line) and $t=2.462\ \alpha$ (short-dashed line).

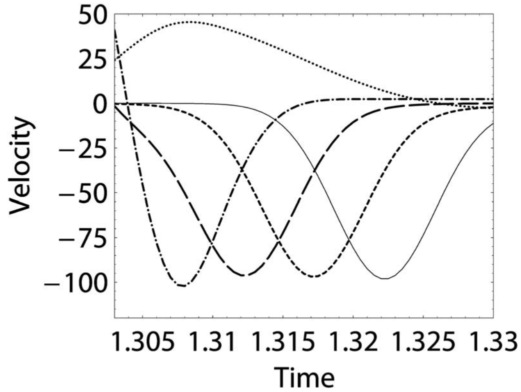

FIG. 5. Velocity of beads (in program units) as a function of time (in units of α) for beads 227 (full line), 228 (short-dashed line), 229 (long-dashed line), 230 (dashed-dotted line) and 231 (dotted line).

To get more insight into this scattering process, let us observe the behavior of the velocity of beads as a function of time. We shall analyze that behavior at both interfaces to see the differences and try to find an explanation for this scattering. To that end we shall fix our attention on those beads around the interfaces.

In Fig. 5 we show the velocity of beads between beads 227 and 231 (i.e., around the right interface) in the time interval 1.303 α and 1.33 α. It can be straightforwardly demonstrated that this behavior is analogous to that found in a system with only two sonic vacua, in case a solitary wave, traveling in an unperturbed medium from the light to the heavy system, scatters from the interface. So it is important to stress that (in the case at hand) in the light system we have solitary waves traveling in it; they correspond (at least) to those leading pulses of the multipulse structure.

In Fig. 6 the velocity of beads 27–30 (i.e., around the left interface) is shown. The big difference between this behavior and the one found at the right interface is notorious. From here we deduce that the origin of the delayed behavior in the scattering process resides in the fact that, contrary to what happens at the right interface, around the left interface beads acquire a constant velocity in the interval $t \in (2.33\alpha, 2.48\alpha)$,

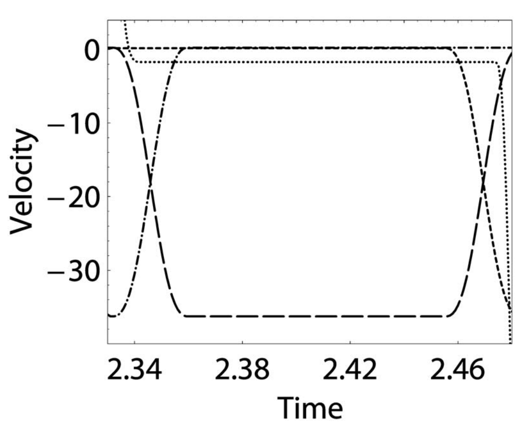

FIG. 6. Velocity of beads (in program units) as a function of time (in units of α) for beads 27 (full line), 28 (short-dashed line), 29 (long-dashed line) and 30 (dotted line).

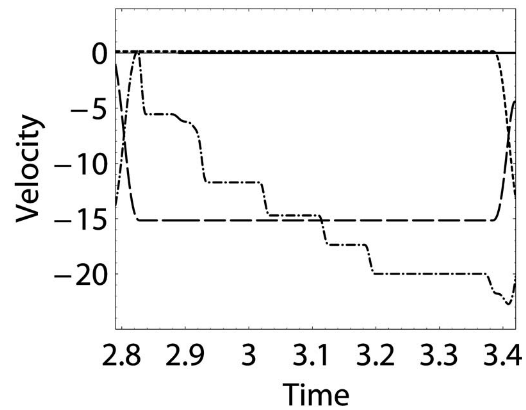

FIG. 7. Velocity of beads (in program units) as a function of time (in units of α) for beads 27 (full line), 28 (short-dotted line), 29 (long-dashed line) and 30 (dashed-dotted line).

where we have previously seen that the transmitted pulse has stopped its movement during a long time. It is also interesting to notice that bead 27 remains at rest during this interval.

When changing the ratio of masses (in this case, the ratio $a:b$) one observes that the interval that takes the transmitted pulse to leave the interface increases. For example, one sees that by keeping the mass and initial velocity of the impacting bead as before, the intervals from the arrival to the left interface of the first backscattered pulse to the instant when the transmitted pulse starts to leave the interface are approximately 0.744 ms, for $a=6$ mm and $b=2$ mm, and 1.244 ms, for $a=8$ mm and $b=2$ mm, respectively (compare with an interval of approximately 0.143 ms for the case $a=4$ mm and $b=2$ mm. A numerical experiment with $L=50$ beads in the interior of the container shows that this time interval does not depend on L).

In case $a=6$ mm and $b=2$ mm, Fig. 7 shows the behavior of the velocity of beads for beads 27–30 in the interval $t \in (2.8, 3.392)$ α, where the transmitted pulse remains "frozen" at the interface. Notice that such behavior for beads 27, 28 and 29 is essentially the same as the one observed in Fig. 6 for the same beads (in particular, bead 27 remains at rest). This confirms that the reason for the behavior observed in this work resides in the fact that some beads near the interface acquire a constant velocity during the interval where the transmitted solitary wave retards its movement. Now, a constant velocity means that no forces are acting on them and therefore the phenomenon of gaps opening occurs.

Using a detailed numerical approach, we have studied the scattering of solitary waves in a granular container consisting of three sonic vacua with Hertzian contact. We have found that the scattering process is elastic at the second interface, while at the first interface it is observed that the transmitted solitary wave has stopped its movement during a time that gets longer when the ratio between masses at the interfaces increases. At the same time, the reflected pulses appear to scatter elastically from the first interface. The opening of gaps in the vicinity of the left interface plays a crucial role in the observed behavior. The understanding of this kind of behavior may be of help for applications of energy-trapping granular containers.

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